

# Multilevel modeling for eye tracking data

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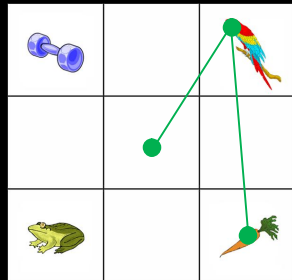
Moss Rehabilitation Research Institute

## Overview

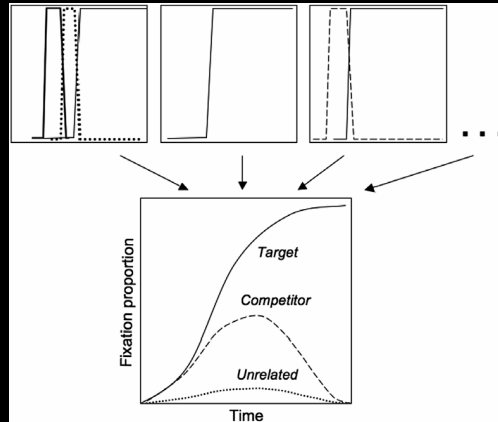
- “ Conceptual Foundations
  - . Eye tracking data, research questions
  - . Why use multilevel modeling?
  - . What is multilevel modeling?
    - “ Conceptual description
    - “ Some **MATH**
- “ Walkthrough (afternoon)
- “ Advanced topic: Individual differences (tomorrow)

# Eye tracking data

Measuring the **time course** of language processing

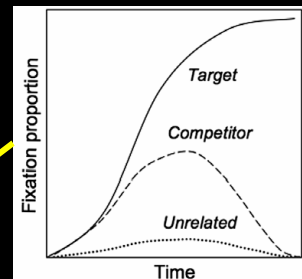
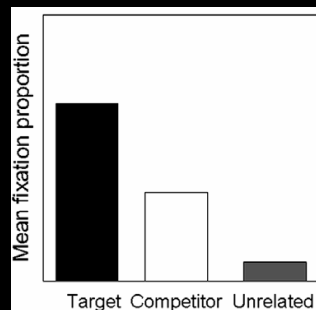


“carrot”



# Research questions

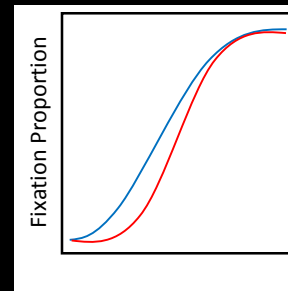
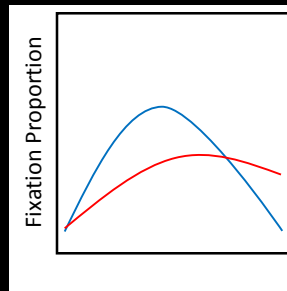
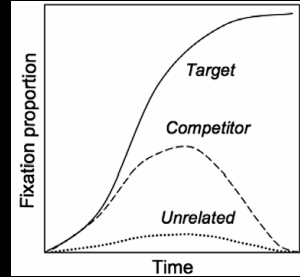
1. Are conditions different?



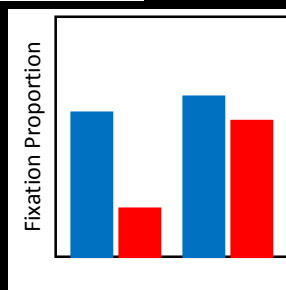
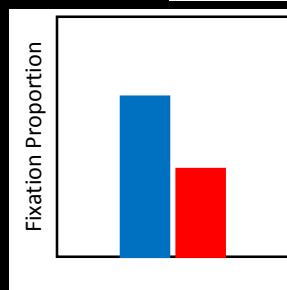
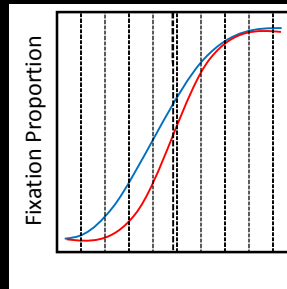
# Research questions

1. Are conditions different?
2. When are they different (or not)?
3. How are the time courses different?

“BLUE is faster than RED”

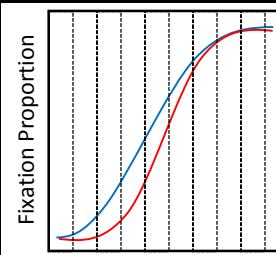
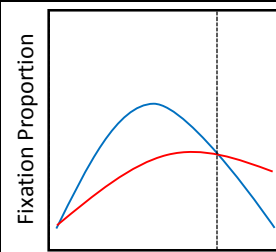


# Time Windows



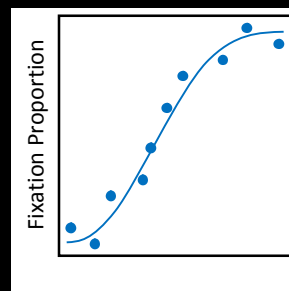
## Problems

- “ What are the “correct”, experimenter-independent time windows?
  - Especially for cross-over effects
- “ Continuity: time bins lose the relationship between time windows
- “ Power: each window has relatively little data

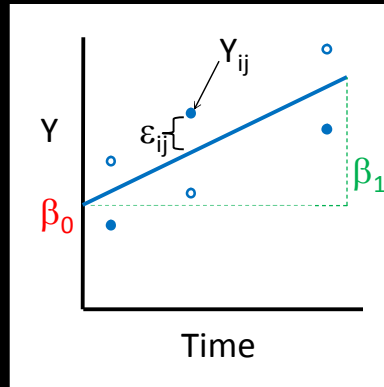


## The Alternative: Regression

- “ Continuous time
- “ Effects of covariates
- “ Not limited to linear effects
- “ Quantify subject/item variability
- “ More flexibility about distribution assumptions



## A simple example



$\beta_0$  : Intercept  
 $\beta_1$  : Slope  
 “Structural”  
 “Fixed” Effects

$\epsilon_{ij}$  : residual error  
 from individual  
 observations  
 “Stochastic”  
 “Random”  
 “Residual” Effects

$$Y = \beta_0 + \beta_1 * \text{Time}$$

$$Y_{ij} = \beta_{0i} + \beta_{1i} * \text{Time}_j + \epsilon_{ij}$$

$$Y_{ij} = \beta_{0i} + \beta_{1i} * \text{Time}_j + \epsilon_{ij}$$

Fixed effects

Random effects

- “ Unique intercept ( $\beta_{0i}$ ) and slope ( $\beta_{1i}$ ) for each participant
- “ Multiple values of Time for each participant
- “ Assuming same functional form for each participant

- “ Unique residual ( $\epsilon_{ij}$ ) for each observation (Participant x Time)
- “ Represents measurement error and “unexplained variance”
  - “There are other predictors”
- “ Distributional assumptions: residuals drawn from normal distribution with mean 0

# Multi-level modeling

## Level-1

$$Y_{ij} = \beta_{0i} + \beta_{1i} * \text{Time}_j + \varepsilon_{ij}$$

## Level-2

$$\beta_{0i} = \gamma_{00} + \zeta_{0i}$$

$$\beta_{0i} = \gamma_{00} + \gamma_{0c} * C + \gamma_{0i} * P_i + \zeta_{0i}$$

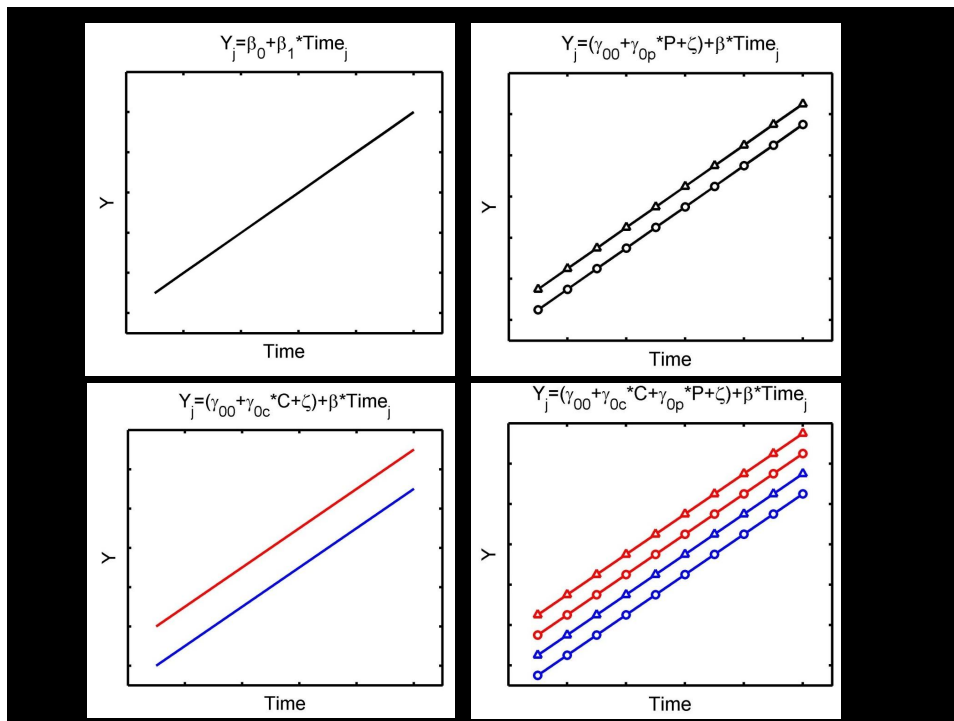
" Model of the Level-1 parameter ( $\beta_{0i}$ )

"  $\gamma_{00}$  = population mean

"  $\zeta_{0i}$  = individual deviation from mean

"  $\gamma_{0c}$  = fixed effect of condition C on intercept

"  $\gamma_{0i}$  = fixed effect of Participant i on intercept



## Multi-level modeling: Residual error

Level-1  $Y_{ij} = \beta_{0i} + \beta_{1i} * \text{Time}_j + \varepsilon_{ij}$

Level-2  $\beta_{0i} = \gamma_{00} + \gamma_{0c} * C + \gamma_{0i} * P_i + \zeta_{0i}$   
 $\beta_{1i} = \gamma_{10} + \gamma_{1c} * C + \gamma_{1i} * P_i + \zeta_{1i}$

Residual error ←

“  $\zeta_{0i}$  = unexplained variance in intercept

“  $\zeta_{1i}$  = unexplained variance in slope

“ Unexplained variance → individual differences

“ Require a lot of data to estimate

## Modeling time

“ Fixation curves are not straight lines

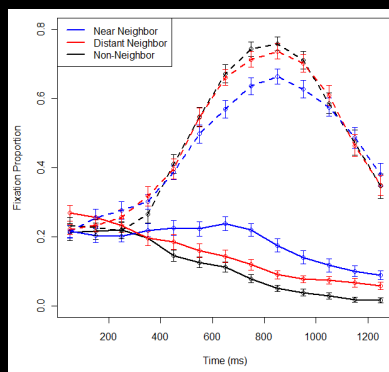
“ Higher-order polynomials

➤ **Cons:** bad at capturing asymptotic behavior

➤ **Pros:** (1) can model any curve shape, (2) dynamically consistent

“ How to choose polynomial order?

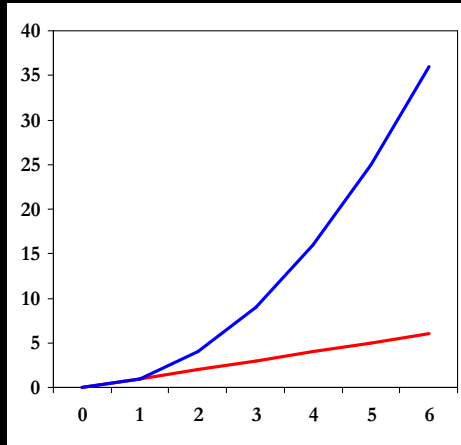
1. **Statistical:** include only terms that statistically improve model fit
2. **Theoretical:** include only terms that are predicted to matter
3. **Orthodox:** use 4<sup>th</sup> order polynomials because I said so



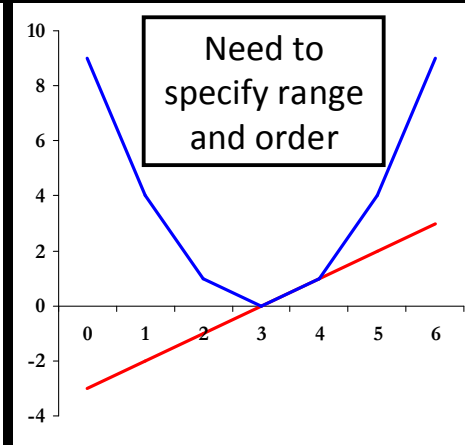
A useful transformation:  
Orthogonal polynomials

# Natural vs. Orthogonal polynomials

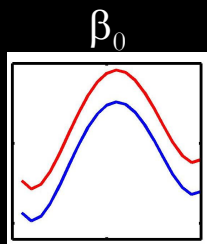
Natural Polynomials  
Correlated time terms



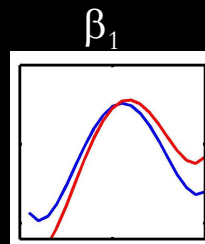
Orthogonal Polynomials  
Uncorrelated time terms



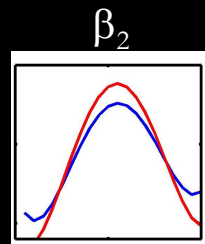
# Interpreting orthogonal polynomial terms



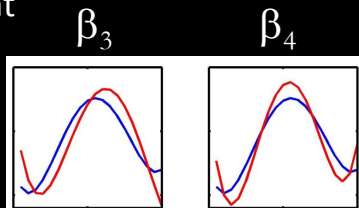
Average  
curve height



Overall slope



Centered rise  
and fall rate



Inflection  
steepness



## Interpreting condition-x-time effects

$$Y_{ij} = \beta_{0i} + \beta_{1i} * \text{Time}_j + \beta_{2i} * \text{Time}_j^2 + \beta_{3i} * \text{Time}_j^3 + \beta_{4i} * \text{Time}_j^4 + \varepsilon_{ij}$$

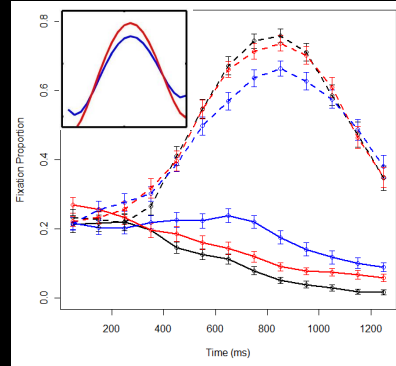
$$\beta_{2i} = \gamma_{20} + \gamma_{2c} * C + \gamma_{2i} * P_i + \zeta_{2i}$$

“  $\gamma_{2c}$  = unique quadratic term for condition C

➤ Relative steepness for condition C

“  $\gamma_{2i}$  = unique quadratic term for Participant i

➤ Relative steepness for Participant i



## Maximum Likelihood Estimation

- “ Find an estimate of parameters that maximizes the likelihood of observing the actual data
- “ Unlike OLS, not a closed-form solution
- “ But very powerful and much more flexible
- “ Goodness of fit measure: log likelihood (*LL*)
  - Not inherently meaningful (unlike  $R^2$ )
  - Change in *LL* indicates improvement of the fit of the model
  - Changes in  $-2LL$  (aka “Likelihood Ratio”) are distributed as  $\chi^2$ 
    - “ Requires models be nested (parameters added or removed)
    - “ DF = number of parameters added

## Conceptual summary

- “ Challenges of longitudinal data
- “ Multilevel modeling
  - Orthogonal polynomials
  - Fixed effects
    - Evaluate effects of experimental manipulations
  - Random/residual effects
    - Evaluate individual differences
  - Continuous covariates
- Next: gaze, attention, and the visual world paradigm
- Afternoon: Conceptual and hands-on walkthroughs