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Multifractal Dynamics in the Emergence of Cognitive Structure

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Abstract

The complex-systems approach to cognitive science seeks to move beyond the formalism of information exchange and to situate cognition within the broader formalism of energy flow. Changes in cognitive performance exhibit a fractal (i.e., power-law) relationship between size and time scale. These fractal fluctuations reflect the flow of energy at all scales governing cognition. Information transfer, as traditionally understood in the cognitive sciences, may be a subset of this multiscale energy flow. The cognitive system exhibits not just a single power-law relationship between fluctuation size and time scale but actually exhibits many power-law relationships, whether over time or space. This change in fractal scaling, that is, multifractality, provides new insights into changes in energy flow through the cognitive system. We survey recent findings demonstrating the role of multifractality in (a) understanding atypical developmental outcomes, and (b) predicting cognitive change. We propose that multifractality provides insights into energy flows driving the emergence of cognitive structure.

Keywords: Fractals; Multifractals; Diffusion; Emergence; Power laws

1. Introduction

Recent evidence suggests that cognition is characterized by seamless interactions among multiple scales of organization (e.g., chemical, physiological, and environmental). These

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interactions support richly flexible and context-dependent behavior, as well as the highly structured and stable phenomena that are typically the focus of cognitive science, such as memory, categories, and the like. A fundamental challenge for cognitive science is to develop a formalism allowing us to understand how the broad spectrum of cognitive phenomena emerge from multiscale organization. In this article, we propose that the methods of fractal scaling provide a ready candidate formalism. More important, physical phenomena displaying fractal scaling provide a viable bridge between behavioral phenomena and the energy flow within the cognitive system (Stephen, Boncoddo, Magnuson, & Dixon, 2009).

We begin by addressing a foundational issue regarding the architecture of cognition: Does cognition arise from the activity of insular components or the multiplicative interactions among many nested structures? Evidence from a wide range of domains suggests that interactions dominate cognition, raising important questions about the viability of approaches employing additive decomposition. Next, we survey how a central measure of interactivity, the power-law exponent, has been fruitfully employed in two different areas of research: atypical developmental populations and emergence of new cognitive structure. Research comparing typically and atypically developing children across a variety of tasks has revealed qualitative differences in interactivity. We then introduce recent findings in which changes in the power-law exponent presage the emergence of new cognitive structure. Finally, we outline how changes in fractal scaling motivate a more general, multifractal approach that may quantify the energy flow across the multiscale structures supporting cognitive performance. Our discussion focuses on conceptual topics; technical details are available in the original articles. Given the constraints of this forum, our survey of the literature is necessarily selective, rather than comprehensive.

2. Power-law relationships throughout the cognitive system

Cognitive performance reveals an interesting mix of stability and instability. For example, cognitive structures, such as those involved in memory and categorization, are conventionally defined by their temporal stability. However, sufficiently detailed measurements show that cognitive performance fluctuates, from memory retrieval and reaction times to syllable durations, acoustical power of vocalizations, and movements of hand and eye. At first glance, these fluctuations may appear uninteresting or even bothersome. Indeed, conventional treatments of cognition assumed that these fluctuations are akin to measurement error. Equating intrinsic fluctuations and measurement error allowed for the adoption of standard linear statistical models. By employing these models, scientists assume that fluctuations in cognitive performance (i.e., variances) are additive, unsystematic white noise (e.g., uncorrelated and normally distributed). Componential theories of cognition go hand in hand with statistical models assuming additive white noise (Sternberg, 2001; Wagenmakers, Farrell, & Ratcliff, 2004). In these well-known treatments of cognition, the internal functioning of each component is insulated from the rest of the system. Each component takes inputs from other components, but its internal operation is not affected by their states. Functionally independent components make an additive approach to modeling viable. At minimum, componential treatments of cognition require additive, unsystematic fluctuations.

Recent research (Gilden, 2001; Van Orden, Holden, & Turvey, 2003; Stephen, Dixon, & Isenhower, 2009) shows that fluctuations in behavior have a much more complex structure than cognitive scientists have typically recognized. Rather than exemplifying additive white noise, fluctuations in cognitive performance are often closer to ''pink'' noise. Whereas white noise reflects equally sized fluctuations at all time scales, pink noise consists of a fractal decay of fluctuation size with scale: systematically larger fluctuations at longer time scales and smaller fluctuations at shorter time scales. That is, the larger changes in cognitive performance unfold gradually over the longer time scales while cognitive performance at shorter time scales is more stable, and a power law relates fluctuation size to time scale, meaning that the size of change in cognitive performance is proportional to time scale raised to a scale-invariant exponent. Fig. 1 depicts a power-law relationship and illustrates the scale-invariant form of power-law distributed dynamics.

Evidence for fractal scaling is extensive. Pink noise appears in a wide variety of standard cognitive tasks measuring response times, such as simple reaction time, word naming, and lexical decision (e.g., Gilden, 2001; Van Orden et al., 2003). Similarly, judgment tasks, involving temporal and spatial estimation tasks yield pink noise (e.g., Wagenmakers et al., 2004). Explicit daily judgments of self-esteem and implicit measures of racial bias also yield pink noise (Corell, 2008; Delignières, Fortes, & Ninot, 2004). Pink noise is manifest in many repetitive biological and motor functions, such as tapping in synchrony or syncopation with a metronome, interstride intervals in gait, and interbeat intervals of heartbeats (Chen, Ding, & Kelso, 2001; Hausdorff et al., 1997).

Fractal scaling can vary widely. Empirically obtained power-law exponents rarely reflect pink noise (i.e., exponent = 1), and systematic changes in the power-law exponent have theoretically important implications for how performance changes. For example, across a broad

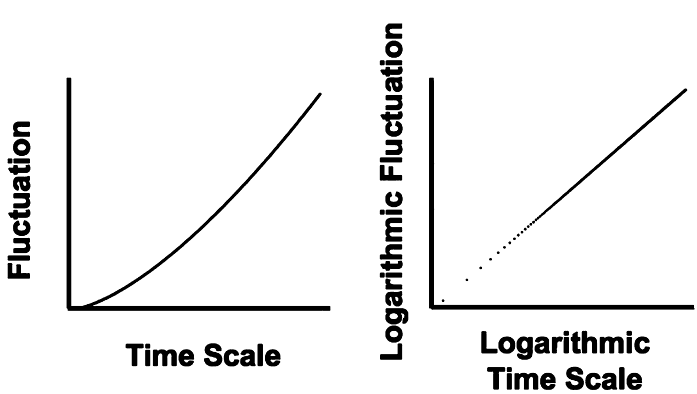

Fig. 1. Power-law relationships depicted on standard (left panel) and logarithmic (right panel) axes. The left panel shows the nonlinear increase in fluctuation size for larger time scales. The right panel shows the scale-invariant property of power laws, that is, that fluctuations increase by the same multiplicative factor across all available ranges of time scales.

range of self-organizing phenomena, the power-law exponent increases to a critical value as a system approaches a phase transition (e.g., Bonamy, Ponson, Prades, Bouchaud, & Guillot, 2006; Heinrich, Levental, Gelman, Janmey, & Baumgart, 2008; Sengers & Shanks, 2009). Thus, a considerable body of work spanning various disciplines (e.g., chemistry, physics, and geosciences) has investigated changes in power-law relationships. For example, injecting unpredictability into a task can weaken pink-noise signals, that is, weaken the relationship between fluctuation size and time scale so that it more closely resembles evidence of additive white noise (Holden, Choi, Amazeen, & Van Orden, in press). This ''whitening'' of a pink-noise signal may reflect weakened interactivity between the cognitive system and the task environment. Comparable examples of changes in power-law relationships can be found in physiological development over the longer term. For instance, pink noise in the timing of strides in gait will whiten with age and with the development of neurological disorders such as Huntington's disorder (e.g., Hausdorff et al., 1997; Van Orden, Kloos, & Wallot, 2009).

Power-law distributions in response times reveal crucial aspects of cognitive architecture (Rhodes & Turvey, 2007; Moscoso del Prado, unpublished data). Assumptions underpinning conventional approaches, specifically that cognitive architectures comprise functionally independent components (e.g., Sternberg, 1969, 2001), do not predict power-law distributions. Whereas normal distributions reflect variability produced by summing together contributions from independent sources, power-law distributions arise naturally in the context of highly interactive, interdependent systems (Bak, 1996; Jensen, 1998). Thus, power-law distributed response times suggest that interactivity and interdependence are inherent properties of cognitive performance (Holden, Van Orden, & Turvey, 2009). Next, we consider how changes in power-law relationships provide a valuable window on cognition. Specifically, we first discuss how differences in power-law relationships may inform our understanding of atypical cognitive outcomes. We then illustrate how over-time changes in the power-law exponent predict a cognitive transition, from one representation of a problem to another.

3. Changes in power-law distributions: Interactivity in neuropsychology

Component-dominant perspectives remain at the core of most modern neuropsychology. This is most clear in the logic of dissociations: When an individual's performance on task *A* is impaired, but performance on task *B* is not, this is taken as evidence that the component responsible for performance of task *A* is impaired. This approach is not without critics (e.g., Van Orden, Pennington, & Stone, 2001) and there is a rich philosophical literature on its strengths and weaknesses (e.g., Caramazza, 1986; Dunn & Kirsner, 2003); however, for present purposes the critical issue is that dissociations are framed in terms of functionally independent components.

For example, patients exhibiting difficulty naming common objects from pictures have been divided into two groups: those with impaired semantic knowledge and those with impaired access to semantic knowledge (e.g., Jefferies & Lambon Ralph, 2006; Warrington

& Cipolotti, 1996). On this interpretation, there is a ''semantic knowledge'' component and a ''semantic access'' component that can be damaged separately. Another key example is the dissociation between surface dyslexic patients, who can correctly read nonwords (e.g., ''bint'') but are impaired on reading exception words (e.g., ''pint,'' incorrectly regularized to rhyme with ''mint''), and deep dyslexic patients who show the opposite pattern of impairment (impaired reading of nonwords and somewhat spared ability to read exception words). Notwithstanding an active debate on the interpretation of these data, the two primary accounts both explain the dissociation in terms of a division of labor between phonological and lexical-semantic components.

Accounts based on differences in processing dynamics begin to suggest interaction-dominant alternatives to component-based accounts of deficits. For example, Gotts and Plaut (2002) used computational model simulations to show that subtle changes in system-wide processing dynamics could produce the behavioral pattern associated with impairment of semantic access. Similarly, Kello and colleagues (Kello & Plaut, 2003; Kello, Sibley, & Plaut, 2005) showed that manipulation of a single parameter of processing dynamics can produce the double dissociation between surface and deep dyslexia.

Some disorders have a more varied set of symptoms, making componential accounts somewhat more difficult. For example, children with autism spectrum disorders (ASD) exhibit impairments or abnormalities at all levels of cognitive function: from perception and action, to language and communication, to social interaction. In addition, deficits in any one component may be able to account for the full range of symptoms. For example, a low-level perceptual bias in favor of local features over global shape processing (e.g., Behrmann, Thomas, & Humphreys, 2006) would predict an impairment of face processing, which could produce an impairment of social interaction and related communication deficits. On the other hand, an impairment of orienting to socially relevant stimuli (e.g., Dawson, Meltzoff, Osterling, Rinaldi, & Brown, 1998) would also predict impaired processing of faces as well as deficits in joint attention and consequent deficits in perceptual development, possibly including general deficits in global shape processing.

Under a component-dominant view of cognition, individual differences should be limited to specific cogs in the cognitive machine. In contrast, an interaction-dominant view of cognition predicts that individual differences should be detectable at all scales of behavior. For example, the dynamics of eye movements should reflect individual differences. Consistent with an interaction-dominant view, the distributions of gaze steps (Euclidean distances between consecutive gaze positions) in language comprehension and visual cognition tasks are best fit by power-law-like distributions (Stephen, Mirman, Magnuson, & Dixon, 2009; Stephen & Mirman, 2010).

Further, task-specific differences in the shape of gaze-step distributions suggest that looking behavior reflects self-organization of the cognitive system in response to task constraints (see also Aks & Sprott, 2003). Differences in the shape of gaze-step distributions also reflect developmental differences. Specifically, in audio-visual speech perception tasks, typically and atypically developing children exhibit eye movements following different kinds of power-law-like distributions. Compared to younger children and children with ASD, older children and typically developing children tended to exhibit distributions of eye movements that more closely resembled a lognormal distribution (Mirman, Irwin, & Stephen, in press). A lognormal distribution is a power-law-like distribution reflecting the multiplicative interaction of independent components (see Holden et al., 2009). It preserves the same interactivity found in pure power-law distributions while reflecting the consolidation of system dynamics into a more stable structure. This difference in eye movements predicted ASD diagnosis beyond standard cognitive and linguistic diagnostic tests and simple measures of eye-movement size. These results suggest that the scale-invariant dynamics of cognition differ across typical versus atypical developmental trajectories, even down to the fine details of eye movements.

Adopting this view requires a shift in thinking about individual differences and neuropsychological conditions. Rather than reflecting differences or deficits in specific components, the behavioral differences reflect differences in system-wide dynamical properties. Such a shift in perspective may help resolve controversies regarding impaired components by reframing the issue in interactive terms. In addition, this approach may allow early detection of neuropsychological conditions through simple noninvasive procedures, such as eye tracking. This is particularly important for developmental disorders (e.g., autism) because early diagnosis can lead to substantially better outcomes.

Historically, efforts to understand individual differences have adopted a strictly component-dominant view, making this field a particularly fertile ground for new insights motivated by adopting an interaction-dominant view on cognitive performance. Work seeking to uncover the systemic, dynamical underpinnings of cognitive deficits can serve as new tests of interaction-dominant views of cognition and may suggest novel methods of diagnosis and rehabilitation.

4. Changes in power-law relationships presage emergence of new structure

Recent work has addressed the emergence of novel cognitive structure from the perspective of interaction-dominant, self-organizing systems using a simple problem-solving task as a test bed (Dixon, Stephen, Boncoddo, & Anastas, 2010; Stephen & Dixon, 2009; Stephen, Dixon, & Isenhower, 2009; Stephen, Boncoddo, et al., 2009). In this task, a set of gear-system problems is presented one at a time. Each gear system consists of a sequence of coplanar, interlocking gears (see Fig. 2) presented in a static display. The turning direction of the first gear is specified by an arrow on its face. The participant is asked to determine the turning direction of the final gear. Across a wide age range, participants usually start solving gear-system problems by manually simulating the rotation of each gear and the transfer of force between adjacent gears. Typically, this force-tracing strategy holds for several trials until, abruptly, in an ‘‘aha!’’ moment, the participant discovers that the interlocking gears form an alternating sequence (e.g., ‘‘clockwise,’’ ‘‘counterclockwise’’) and use that property to solve the problem. The sudden, spontaneous discovery of a new relationship, alternation, marks the emergence of a new cognitive structure, by all conventional accounts, a new representation of the problem (Dixon & Bangert, 2002).

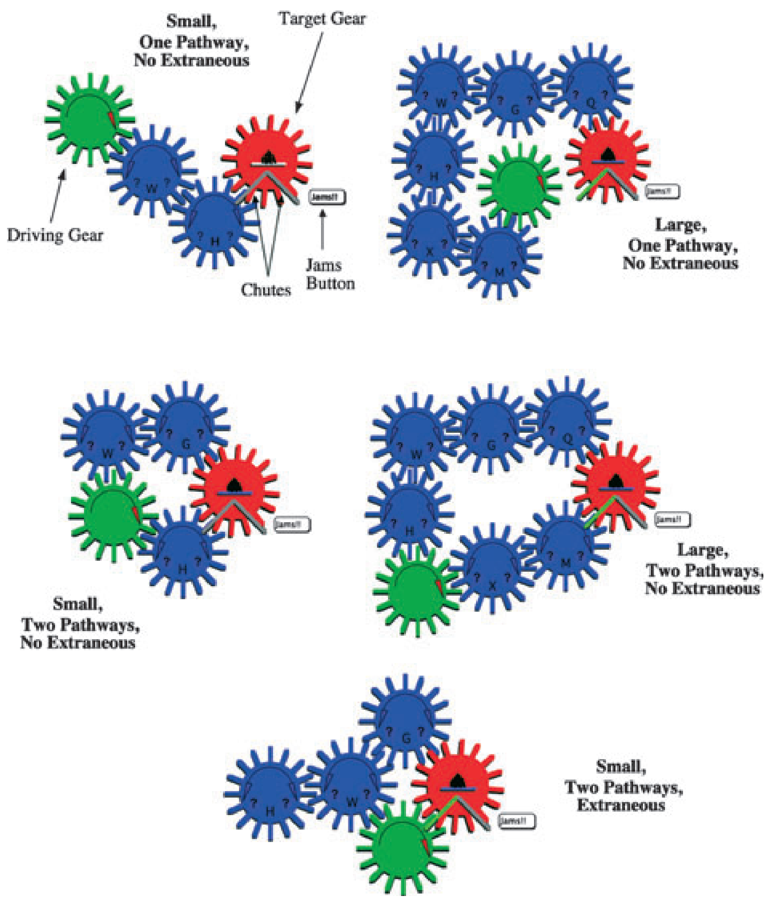

Fig. 2. Examples of gear-system problems, varied on dimensions of size (small systems with 4–5 gears or large systems with 7–8 gears), number of pathways (1 or 2), and presence or absence of an extraneous gear.

If interactions dominate the cognitive system, then the emergence of a new cognitive structure should have the properties of a phase transition, a sudden qualitative change in the organization of the system that arises from a critical instability, the breaking and reforming of componential constraints on the system. This temporary weakening of componential constraints occasions a concomitant temporary strengthening of interactions, marked by peak and subsequent drop in the power-law exponent (relating fluctuations to time scales; e.g., Grebogi, Ott, Romeiras, & Yorke, 1987). To test the hypothesis that the discovery of this new cognitive structure was a phase transition, we needed very fine-grained data on performance during the task. We therefore asked participants to wear a motion-tracking device on their dominant hand while they were solving the gear-system problems. For each trial, the time series of angular velocities was calculated. The angular velocity time series were in the range of pink noise. More important, participants who discovered alternation showed a peak and subsequent drop in their power-law exponents just prior to discovery. Participants who did not discover alternation showed no significant change in their power-law exponents. In subsequent work, Stephen, Boncoddo, et al. (2009) extended these findings using a time series derived from eye tracking rather than hand movements. They found that a peak and subsequent drop in the power-law exponent anticipated discovery of the new representation. Regardless of whether the time series was obtained from the hand or the eyes, the transition to a new cognitive structure was predicted by a peak and subsequent drop in the power-law exponent.

Structures called ‘‘representations’’ may more accurately be described as functional organizations of the cognitive system. Functional organizations are not committed to specific components. In the gear task, hand or eye may both serve to attune the cognitive system to the energy gradients entailed by the task environment, and fluctuations in each exhibit the same pattern of power-law exponents. This organization is not reducible to anatomical components but rather functional relationships occasioned by energy flow (e.g., Kugler & Turvey, 1987).

5. Multifractality as a formalism for the energy flows underlying information

In a component-dominant system, a change in behavior can be straightforwardly attributed to a particular source. The individual components form a causal chain, so identifying which component or set of components is responsible for change simply requires analyzing the causal connections in the chain. Explaining stability in such systems is not typically an issue because the components are implicitly assumed to have temporal stability. Since the output of one component is an input to the next component in the chain, it often makes sense to address these between-component transactions as information exchange.

In an interaction-dominant system, changes in behavior have a very different source, because in such a system behavior is the result of transactions among very many components. The flow of energy across a complex web that constitutes the components gives rise to behavior. These components are not arranged in a causal chain, nor do they function independently of one another. The minimal crucial difference between component-dominant and interaction-dominant systems is that in an interaction-dominant system the internal functioning of each component is dependent on the functioning of other components. Behavior in an interaction-dominant system is a macroscopic phenomenon emerging from the interactions among all the components. Given that behavior is softly and temporarily assembled in interaction-dominant systems, stability is a phenomenon of interest, just as change is. Indeed, they are governed by the same principles (Shinbrot & Muzzio, 2001). Put simply, behavior is just too flexible, adaptive, and context sensitive to be plausibly driven by a formalism rooted in the physical mechanics of strictly linear Newtonian systems (Van Orden et al., 2003; Stephen, Boncoddo, et al., 2009). Whereas Newtonian systems are built of smooth Euclidean components, fractal systems are rife with multiscale imperfections (i.e., fluctuations) that can radically change outcomes (Stewart & Golubitsky, 2001). By definition, emergent behavior does not reduce to the sum of behavior of the component processes composing the system (Van Orden, Holden, & Turvey, 2005). Because the collective interactions across the components give rise to emergent behavior, as opposed to the activity of any one component, the notion of information exchange is poorly suited to interaction-dominant systems.

Because information exchange is not a sensible metaphor for interaction-dominant systems, we pursue a formalism in terms of energy. At first glance, this may appear to be a radical departure, but note that information exchange in the cognitive system must be a generically physical (and hence energy-consuming) set of events (unless one wishes to accept dualism; e.g., Descartes, [1641/1998]). Thus, addressing cognition in terms of the energy flow is a broader, more general approach, one that will include activity typically considered to be information exchange. In what follows, we explain how fractal scaling connects cognition seamlessly with the flow of energy and matter across the system. We then locate changes in fractal dimension within the broader formalism of multifractality.

Power-law relationships represent a special case of diffusion. Diffusion is the movement of physical material in a medium. Importantly, diffusion only occurs when there is a gradient of energy or matter within the medium. Traditionally, models of diffusion have begun at the level of the motion of a single particle. This motion is quantified in terms of the mean squared distance (MSD) covered by the particle as a function of time. In ordinary Newtonian models of diffusion, average squared distance increases as a linear function of time. Power-law relationships emerge when MSD increases faster than a linear function of time (time to the power of 1) but not faster than a quadratic function of time (time to the power of 2). Because of the *fractional* exponent on time, between the whole numbers 1 and 2 is often called fractal diffusion (Scafetta & Grigolini, 2002; Shlesinger, Zaslavsky, & Klafter, 1993). Fractal diffusion (i.e., diffusion in the power-law range) occurs in complex physical media in which the gradients of energy and matter are heterogeneous.

What does diffusion have to do with cognition and behavior? Cognition, because it involves the activity of physical components, must consume energy. Energy consumption will change the local gradients of energy and matter, and therefore the speed at which energy flows through the system, that is, the rate of diffusion. Thus, the activity that entails cognition must change the rate of diffusion in the complex physical materials in which it occurs. Whereas the classic diffusion model of response time (e.g., Ratcliff, 1978) dealt with modeling accrual of evidence in a component-dominant cognitive architecture, the present discussion recommends empirical estimation of diffusion rates as a means to predict cognitive outcomes.

Regardless of whether we view behavior as intrinsically meshed with cognition (e.g., Kaschak et al., 2005) or just tightly time-locked to cognition (e.g., Magnuson, Tanenhaus, Aslin, & Dahan, 2003), it follows that fine-grained measurements of behavior carry information about the diffusion rates of the structures generating it. The fluctuations emphasized above are the source of this information about diffusion rates. Fluctuations in macroscopic behavior are the aggregated, gradient-dependent movements of material across many scales of the cognitive system that support the behavior. The power-law exponent relating the magnitude of fluctuations to the time scale quantifies the rate of diffusion.

In very complex material, such as biological tissue, diffusion is likely to occur at different rates across different scales. Here, a single power-law exponent will not suffice to describe energy flow through the system. Rather, we need a range of power-law exponents to describe the spectrum local rates of energy flow within the system (e.g., Aranda, Salgado, & Munoz-Diosdado, 2006). Whereas diffusion depending on a single fractional exponent relating MSD to time is fractal, diffusion that depends on multiple fractional exponents relating MSD to time is multiply fractal, or more simply termed multifractal (Shlesinger et al., 1993).

Multifractal diffusion expresses energy flow across a much broader range than what cognitive science has traditionally taken as the scope of information transfer. Information transfer may only address a subset of the energy flow that the cognitive system uses to organize to its environment. In short, no fluctuation may be too small or too large to exert a meaningful effect on cognitive performance. Recent work has demonstrated that aspects of cognitive performance are indeed multifractal (Ihlen & Vereijken, 2010; Stephen & Dixon, 2011). For example, Stephen and Dixon asked participants to tap in synchrony with an unpredictable (i.e., chaotic) metronome. They showed that the intertap intervals participants generated were multifractal, exhibiting a spectrum of fractal scaling exponents within the same intertap interval time series. The multifractal spectra of these intertap intervals closely matched the multifractal spectra of the corresponding metronome's interonset intervals. These results provide an initial demonstration that a complex behavior (synchronizing to an unpredictable signal) may emerge from the multifractal dynamics characterizing both the cognitive system and its environment.

Fluctuations have traditionally received rather short shrift in cognitive science. However, under a complex-systems perspective, we begin to see that fluctuations characterize the state of an entire cognitive system at very many scales and that the structure of these multiscale fluctuations is centrally relevant to the development of the cognitive system. When we examine the statistical structure of these multiscale fluctuations, it becomes possible to trace out the complex interplay of forces and flows that push and pull the cognitive system as it takes on new structure. In this light, the cognitive system shows itself to be a physical system built of components that interact similarly at very many scales. Such systems are necessarily structured by the power-law structured flow of energy, indicating nonlinearly fast diffusion. We propose that it is possible to understand and predict the development of such systems by empirically estimating the changes in power-law structure of the system behavior. The multifractal dynamics of the fluctuations in the cognitive system provide complex-systems approaches to cognitive science with new leverage on solving the problem of how a cognitive system evolves.

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