



## Gaze fluctuations are not additively decomposable: Reply to Bogartz and Staub

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### ABSTRACT

Our previous work interpreted single-lognormal fits to inter-gaze distance (i.e., “gaze steps”) histograms as evidence of multiplicativity and hence interactions across scales in visual cognition. Bogartz and Staub (2012) proposed that gaze steps are additively decomposable into fixations and saccades, matching the histograms better and illustrating how additive processes can generate tailed histograms. In this reply, we consider the validity of fixation-versus-saccade distinctions, reviewing eye-movement literature and re-analyzing our original data. Careful examination of empirical literature undermines rigid fixation-versus-saccade distinctions. By comparing original gaze-step series with surrogate data, we present new evidence that temporal clustering in gaze-step data reflects interactive rather than additive processes. We conclude by discussing the relation between traditional notions of interactivity between components and complex-systems notions of interactivity across scales.

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### 1. Introduction

A central question in cognitive science is whether cognitive systems are composed of distinct, independent processing components or whether their performance reflects interactions across many scales. That is, not only may components interact on a strictly cognitive scale of processing, but the activity of each component may depend on its nesting, within larger components and the context at large. This question has been investigated for decades through a variety of clever manipulations of tasks and stimuli using traditional experimental psychology paradigms (for a recent review see Mirman, Bolger, Khaitan, & McClelland, *in press*). More recently, insights from statistical physics have provided new tools for examining perceptual-motor fluctuations for evidence of interactions across scales in

cognitive systems (Holden, Van Orden, & Turvey, 2009; Ihlen & Vereijken, 2010; Van Orden, Holden, & Turvey, 2003, 2005). One link between statistics and interactivity is the mathematics of random factors: the sum of very many independent factors typically yields normal distributions over time, and the multiplication of interacting factors across many scales can yield heavy-tailed distributions over time (e.g., lognormal or power law; Sornette, 2004). For example, Holden et al. (2009) argued for interactivity based on heavy-tailed (lognormal and power law) distributions of response times in naming and reading tasks.

Extending this work, we (Stephen & Mirman, 2010) examined the distribution of pixel distances between sampled gaze positions on a computer screen (henceforth, “gaze steps”) during three visual-cognitive tasks. Like Holden et al.’s response-time data, gaze-step histograms were heavy-tailed. We compared these empirical histograms to best-fitting (according to maximum likelihood estimation) normal, exponential, gamma, lognormal, and power-law distributions, assuming only one degree of freedom rather

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than testing mixtures of these distributions. The best-fitting one-degree-of-freedom distribution was lognormal, with only one participant's gaze steps in one task exhibiting power-law form. Following Holden et al.'s reasoning, this evidence was consistent with the hypothesis that visual cognition reflects interactions among multiple scales of behavior.

## 2. Criticism by Bogartz and Staub

Bogartz and Staub (2012; henceforth "B&S") raised two important concerns regarding this conclusion:

- (1) Our modeling of gaze steps failed to address cognitive, neural, and motor processes underlying visual cognition. Gaze is the sum of two separate mechanisms: fixations and saccades.
- (2) Considering the overall distribution of gaze steps ignores the temporal structure of the gaze steps: gaze steps are clustered in periods of very little positional change (fixations) and periods of rapid change (saccades).

Addressing the first concern, we review the eye-tracking literature highlighting that, although serving as a useful descriptive heuristic, a fixation-versus-saccade dichotomy fails to capture a broader, more continuous variety of eye movements. Further, we will show that B&S's additive two-process model leads to implausible predictions.

The second concern applies to any analysis omitting time – our original model and B&S's model. According to B&S, the temporal clustering of gaze steps corresponds to two separate states – fixations and saccades. However, there exist analytical methods for directly testing whether temporal structure is due to purely additive processes (i.e., linear autocorrelation, described below) or to interactive processes. We will show that the temporal structure in gaze-step time series is consistent with interactive rather than additive processes, consistent with our original conclusion (Stephen & Mirman, 2010).

## 3. Empirical evidence for fixations and saccades as two separable states

As B&S note, gaze behavior in visual search tasks like the ones we tested is usually described in terms of two states: fixations and saccades. However, it is important to remember that this *description* is not a mechanism or model. As Rayner (1998, p. 373) put it, during fixations "our eyes remain relatively still", but, critically, he went on to point out that "the term *fixation* is something of a misnomer. The eyes are never really still". He further identified three non-saccadic eye movements – nystagmus, drift, and microsaccades – which "most experimenters interested in reading assume [...] are "noise" and adopt scoring procedures that ignore them" (p. 374). Indeed, these "fixational" eye movements appear to be influenced by cognitive processes such as covert attention (e.g., Engbert & Kliegl, 2003). The same argument holds for newly-discovered eye movements, such as glissades – wobbling

eye movements at the end of a saccade (e.g., Nyström & Holmqvist, 2010). Strikingly, "researchers must actively choose whether to assign the glissades to saccades or fixations; the choice affects dependent variables such as fixation and saccade duration significantly" (Nyström & Holmqvist, 2010, p. 188). So fixations and saccades not exhaustive, but researchers maintain the illusion that they are by assigning all other eye behaviors to one state or the other.

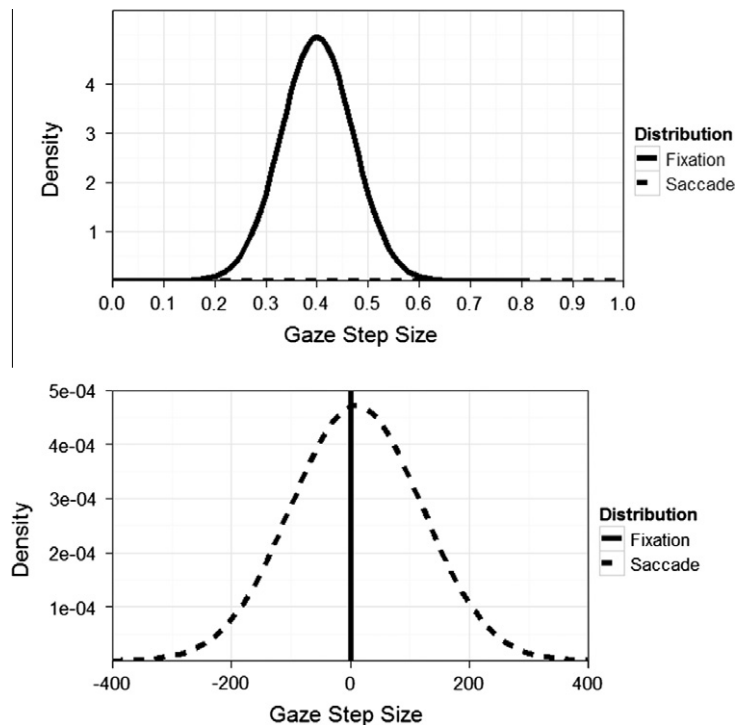
The fixation-versus-saccade dichotomy also belies the practical difficulty of differentiating these two hypothetically distinct states, as illustrated by the proliferation of algorithms for partitioning gaze into fixations and saccades (Karsh & Breitenbach, 1983; Pillalamarri, Barnette, Birkmire, & Karsh, 1993; Salvucci & Goldberg, 2000) and these algorithms' sensitivity to parameters (Blignaut, 2009), display luminance (Doma & Hallett, 1988), visual object type (Manor & Gordon, 2003), and participant population (Shic, Chawarska, & Scassellati, 2008).

Since researchers must actively decide what will count as fixations, saccades, or noise; fixations and saccades more accurately represent idealized endpoints on a continuum, rather than two categorically distinct states. No doubt, for many research questions, separating gaze behavior into fixations and saccades is a reasonable approximation, but approximations should not be mistaken for facts about the underlying system.

## 4. Conceptual problems with argument presented by B&S

Besides the difficulty of distinguishing fixations and saccades empirically, the model that B&S implemented based on this distinction leads quickly to challenges of interpretation. B&S proposed that the addition of two normal (i.e., additive) distributions—one for fixations, the other for saccades—outperforms the lognormal (i.e., a multiplicative) model. However, this additive model entails surprising implications for expected gaze steps. About 20% of the saccade distribution extends beyond 100 pixels (i.e., non-physiological according to B&S) and almost half extends into (logically impossible) negative values (Fig. 1). Hence, B&S omitted most of the saccade distribution, turning a normal distribution into a fragment that fattens the tail of the fixation distribution. As a result, not only may fixations and saccades be difficult to define, but B&S's model is not clearly interpretable.

Where does that leave the B&S account? Their characterization of gaze behavior as the sum of two distinct states is superficial and incomplete; their proposed additive two-state model is actually a one-state-plus-tail-fragment model. Even if their result confirmed that tailed distributions might be fit by additive mixtures of additive distributions, their foundational claim of two distinct states does not match what is known about eye movements and their statistical model does not actually implement the full set of "two additive distributions" proposed. However, their point that gaze steps have temporal structure – that they tend to cluster into periods of relative stability and periods of relatively rapid change – comes with further



**Fig. 1.** Probability density distributions for the additive model proposed by Bogardt and Staub. The solid line represents the fixation distribution; the dashed line represents the saccade distribution. Both panels show the same data. The top panel's axes are scaled to make the fixation distribution most clear (the saccade distribution is the dashed line essentially at Density = 0) and the bottom panel's axes are scaled to make the saccade distribution most clear (the fixation distribution is the vertical line essentially at step size = 0).

entailments for the additivity of gaze steps and brings us to the new question: Does the temporal structure reflect additive or interactive processes? In what follows we describe an approach to answering this question and apply it to our original data.

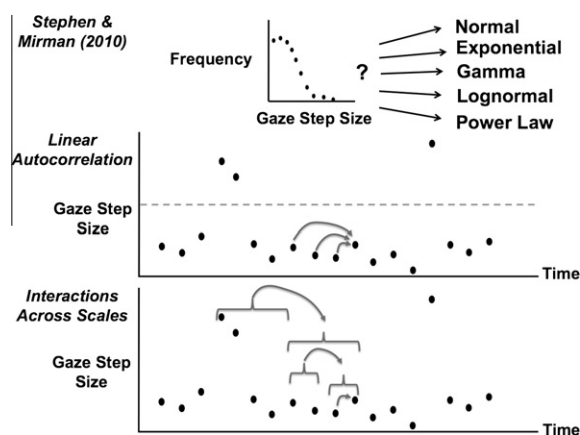
### 5. Temporal clustering: additivity and interactivity in time series

Our original claim and the counter-argument proposed by B&S were based on attempting to match the shape of the aggregate histograms of gaze steps with that of distribution functions. As illustrated in the top panel of Fig. 2, considering aggregate histograms ignores the temporal structure because each point contributes equally to the distribution, regardless of sequence. When we consider the time series – that is, how each particular data point's value fits into the broader sequence of values – we can ask whether the value observed at a particular time reflects linear, additive processes operating over past values or interactive (i.e., multiplicative) processes operating over multiple time scales of past values.

By definition, modeling each value in a purely linear, additive series requires only three parameters: (1) the mean, (2) the variance, and (3) the linear autocorrelation (Theiler, Linsay, & Rubin, 1994). In plainer terms, this definition of additive process means that each and every value of an additive process must be well predicted by (1) the average of all values in the series, (2) the square root of

the average squared differences of all values from the mean, and (3) the average independent contributions of each previous value to each current value (Fig. 2, middle panel). Linear autocorrelation is a necessary mathematical entailment of B&S's view of temporal clustering as the addition of two separable processes. Roughly speaking, short-lag positive correlations produce clusters of similar-valued gaze steps and longer-lag negative correlations produce alternation between “fixation” clusters and “saccade” clusters. If temporal structure reflects interactions across time scales (Fig. 2, bottom panel), gaze-step series cannot be exhaustively modeled by the mean, variance, and autocorrelation. In this multiplicative case, each gaze step would not be predictable only as part of a single fixation or saccade but instead as the result of extended sequences of fixations and saccades preceding the current value over many time scales at once.

We can statistically test whether an observed series fits the mathematical definition of additivity, i.e., whether it is exhaustively modeled by its mean, variance, and linear autocorrelations. To do this, we create a distribution of new, “surrogate” versions of the original series, each of which have the same values as the original series (thus preserving the mean and variance) and are constructed to preserve linear autocorrelation (Schreiber & Schmitz, 1996). That is, the average independent contributions of previous values to current values can mimic those found in the original series, but the new scrambled order means that the original sequence—and any overlapping of time

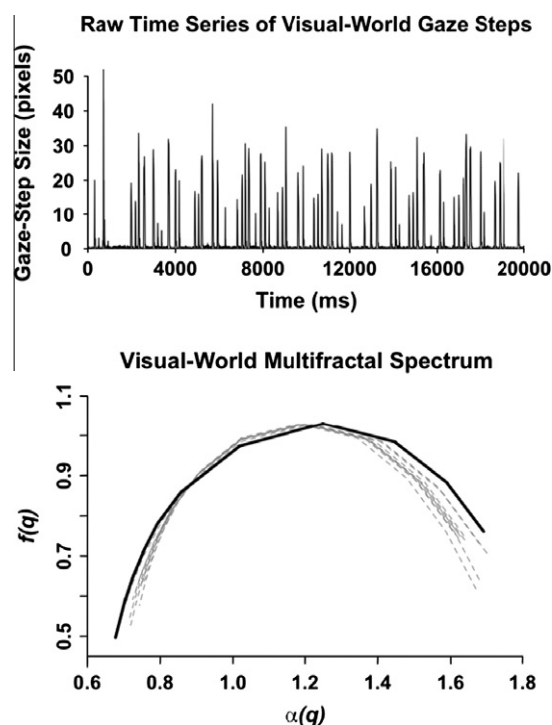


**Fig. 2.** Schematics representing the different hypotheses represented both in our earlier work (Stephen & Mirman, 2010) and in the present reply. The top panel represents our earlier question regarding whether the empirical histograms of gaze steps were best fit by normal, exponential, gamma, lognormal, or power-law distributions in single-degree-of-freedom tests. The middle and bottom panels represent two different hypotheses regarding the exact same schematic example of gaze steps. These two different hypotheses address modeling any given current value of a variable (e.g., the gaze step size for any single given pair of consecutive eye-tracking samples)—we emphasize the 11th value here only for the purpose of the schematic. The middle panel schematizes linear autocorrelation: each previous value contributes, on average, an independent effect on the current value. A grey horizontal line indicates a cutoff that might translate mean gaze-step size into a fixation–saccade distinction, provided Bogartz and Staub’s assumption of additivity. The bottom panel schematizes the multiplicative case in which each single value emerges from the nesting of one time scale within another. Each current value is influenced by the previous value, but at the same time, the value of the current and just previous (lag-1) values reflect influence from the two previous values before them. In turn, these four values together (current, lag-1, lag-2, and lag-3) are influenced by the four values before them—and so on. Whereas linear autocorrelation stresses the independent effects of previous values, multiplicative series exhibit an entangled heredity in which longer-scale behavior constraints progressively shorter-scale behavior.

scales—is destroyed. These new, fabricated additive series are called “surrogates” because they represent the best approximation of original data assuming that these data are additive. If B&S are correct, gaze-step series should be statistically indistinguishable from a sample of additive surrogates. We apply a “multifractal analysis” (Appendix A) to estimate a “multifractal spectrum” and its width. We evaluate whether multifractal-spectrum width for observed gaze-step series deviates significantly from (i.e., falls outside the 95% confidence interval of) multifractal-spectrum widths for a distribution of additive surrogates (e.g., Fig. 3). This comparison provides formal test of a key intuition behind B&S’s argument: that temporal clustering in gaze-step data reflects the addition of two independent processes—fixations and saccades.

## 6. Reanalysis of the gaze-step data

First, we constructed gaze-step series from the first 10,000 gaze steps less than 100 pixels, concatenating across multiple trials of continuous eye-tracking at 500 Hz. Multifractal analysis requires thousands of data



**Fig. 3.** The top panel shows the first 10,000 samples of the gaze-step time series for the visual-world paradigm task for participant 1. The bottom panel shows the multifractal spectrum for the original series (dark, solid curve) as well as those for 10 linear surrogates corresponding to the additive structure of the original series (grey, dashed curve). The multifractal spectrum of the original series here is significantly wider than the multifractal spectra of the surrogates. This observation entails that the temporal sequence of the original series (top panel) is significantly different from what would be expected from purely additive dynamics, that is, from purely separable processes.

points for reliable estimation. Next, we generated 50 additive surrogate series (according to Schreiber and Schmitz’s (1996) IAAFT algorithm) for each participant’s gaze-step series over the course of an entire task.<sup>1</sup> We used Chhabra and Jensen’s (1989) multifractal algorithm for 18 series (i.e., 6 participants in 3 tasks) and for 50 corresponding surrogates (i.e., total of  $18 \times 50 = 900$  surrogates). One-sample, two-tailed *t*-tests contrasted each original series’ multifractal-spectrum width with those of its surrogates (Schreiber & Schmitz, 2000; Fig. 3).<sup>2</sup>

For 15 out of 18 cases (83%), multifractal test rejected pure additivity (schematized in Fig. 2, middle panel) and confirmed multiplicativity (schematized in Fig. 2, bottom panel).<sup>3</sup> These data (Table 1) indicate that, contrary to B&S’s view, temporal structure of the majority of the data under discussion was inconsistent with additivity. Larger samples may provide insights into task or individual

<sup>1</sup> Augmented Dickey–Fuller tests rejected nonstationarity in all cases,  $p < .01$ , suggesting that mean, variance, and autocorrelation were all well-defined for the purposes of constructing the surrogate series.

<sup>2</sup> Bootstrap *t*-tests provided comparable results, with no change in significance.

<sup>3</sup> Pure additivity is the “null hypothesis”; failure to reject is not evidence for additivity.

**Table 1**  
Multifractal spectrum widths for original and surrogate time series.

Participant	VWP			Single-feature search			Conjunction search		
	Original	Surrogates		Original	Surrogates		Original	Surrogates	
		<i>M</i>	<i>SE</i>		<i>M</i>	<i>SE</i>		<i>M</i>	<i>SE</i>
1	1.04**	.98	.006	.71**	.63	.009	1.07**	1.13	.010
2	.88**	.85	.004	.70**	.66	.003	.85**	.94	.007
3	.85	.86	.005	.93	.93	.005	.84*	.86	.006
4	.86	.86	.004	.79**	.76	.005	.79**	.85	.005
5	1.00**	.97	.006	.92**	.79	.005	.94**	.78	.007
6	.97**	.79	.007	.91**	.81	.011	.88**	.83	.008

\*  $p < .01$ .

\*\*  $p < .0001$ .

differences leading to differences in degree of interactivity in different data sets (see also Ihlen & Vereijken, 2010; Mirman, Irwin, & Stephen, 2012).

## 7. Conclusion

Previously, we examined histograms of gaze steps in visual-cognitive tasks to argue for interactions across multiple scales in the visual cognitive system (Stephen & Mirman, 2010). Bogartz and Staub (2012) stressed that using gaze steps ignores decades of eye tracking research showing that, in the tasks we tested, gaze transitions between two states: fixations and saccades. However, closer examination of the literature shows that this distinction is only a heuristic approximation; gaze actually exhibits movements varying widely across individuals and contexts. This context-sensitivity and variability is typical of systems structured by cross-scale interactivity. Further, although Bogartz and Staub's sum-of-two-normals model statistically outperformed our single-lognormal distribution, this two-state model makes deeply questionable assumptions about the distribution of saccades. So, not only were B&S's theoretical assumptions questionable, their statistical model requires arbitrary tailoring of distributions to implement those assumptions.

Bogartz and Staub also pointed out that gaze steps appear to be temporally clustered in periods of relative stability and periods of rapid movement. They took this clustering as evidence of two distinct states summed together, but closer inspection shows that this additive description is inaccurate. Multifractal analysis of the time series showed that temporal clustering was mostly inconsistent with additive models in which the eyes alternate between independent states of fixations and saccades. Gaze steps are indeed clustered in time, but the clustering goes beyond alternating between two states, it reflects nested processes unfolding at many different time scales simultaneously.

We agree with Bogartz and Staub that the question of interactivity is just about as old as cognitive science itself. Historically, cognitive scientists have asked the interactivity question with regard to particular levels of processing (Dell, 1986; Rumelhart, 1977, chap. 27); for example, whether there is bi-directional information flow between phonological and lexical processes (for recent reviews see McClelland, Mirman, & Holt, 2006; Mirman et al., in press;

for a recent example of the debate see Farmer, Christiansen, & Monaghan, 2006; Farmer, Monaghan, Misyak, & Christiansen, 2011; Staub, Grant, Clifton, & Rayner, 2009, 2011). If the cognitive system is truly interactive across scales, as we and others have argued and Bogartz and Staub seem to agree (or at least be willing to consider), then the next logical question is: how fundamental are componential distinctions like that between “phonological processing” and “lexical processing”? That is, if phonological processes are influenced by lexical processes and lexical processes are influenced by phonological processes—and if both exhibit effects of context at the scales of population differences, task differences, and stimulus differences, then they are not really separate processes at all.

There are two ways to proceed from this point. The traditional cognitive science way is to assume that the processes are at least somewhat separable and that, to a useful approximation, we can study them separately. This way has led to evidence for interactivity across cognitive and perceptual systems (Mirman et al., in press). A two-factor ANOVA provides the typical model (and the standard analytical tool) for this notion of interactivity: the analysis begins with independent predictors that can contribute alone or in “interaction” with one another. However, because ANOVA assumes independent predictors, this is only the tip of the interactivity iceberg. Newer complexity-based approaches bring with them the suggestion that interactions may not operate solely between separable processes at a given scale of analysis. Rather, interactions may manifest in a spider-web-like entanglement of processes, possibly unfolding seamlessly from relatively large scales to relatively small scales. Analytical approaches from statistical physics and nonlinear dynamics can be used to probe these interactions across scales for insights regarding the emergence, operation, and development of different but closely interrelated, and ultimately inseparable capacities supporting cognition (Stephen, Anastas, & Dixon, 2012).

These approaches need not be in conflict; after all, to some degree, they are aimed at answering the same question and even interactive systems exhibit rich heterogeneous structure. However, it is important to distinguish between theoretical claims and simplifying assumptions. One might make the simplifying assumption that there is, for example, a set of lexical processes that can be studied more or less separately, or that eye movements can be

divided into fixations and saccades for the purposes of studying reading or speech comprehension, but these should not be regarded as fundamental properties of the cognitive system. Heterogeneity of mechanism does not require separability. Raw measurements of perceptual-motor systems may reveal the interwoven structure of the cognitive system in which lexical processes and visual processes reflect effects of intentions, contexts, and tasks at broader scales and effects of motor, physiological, and neural processes at finer scales (Van Orden, Kello, & Holden, 2010). Eye movements are just one measurement we might draw from this swarm of cross-scale interactions, and inventorying separable types of movements is of limited use under such conditions. Much has been learned from decomposing the cognitive system (Bechtel, 2009), but there may be much to learn from approaching the cognitive system instead as a cascade of many inseparable factors.

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### Appendix A

Here, we outline Chhabra and Jensen's (1989) multifractal analysis for non-negative series  $x(t)$  of length  $N$ .

*Step 1:* Bin time series  $x(t)$  using  $N_n$  nonoverlapping windows of length  $4 \leq n \leq N/4$ , where  $N_n$  is the greatest integer  $\leq N/n$ .

*Step 2:* For each  $n$ , calculate proportion  $P$  for  $i$ th bin ( $i = 1, 2, \dots, N_n - 1, N_n$ ),

$$P_i(n) = \frac{\sum_{k=n(i-1)+1}^{ni} x(k)}{\sum_{m=1}^{N_n} x(m)}, \quad (\text{A1})$$

Multifractal analysis examines proportion growth with greater  $n$ —for regions of  $x(t)$  with greater or lesser proportion. For series with homogeneous temporal structure,  $P_i(n)$  is approximately equivalent across  $n$ -sized bins, and proportion should follow the single power-law relationship:

$$\begin{aligned} P(n) &\sim n^1 \\ P(N_n) &\sim N_n^{-1}, \end{aligned} \quad (\text{A2})$$

However, for series with heterogeneous temporal structure,  $P_i(n)$  may vary considerably across bins, growing more slowly or more quickly with  $n$  for regions of  $x(t)$  with higher or lower proportions, respectively. For series with heterogeneous temporal structure, Eq. (A2) generalizes to

$$\begin{aligned} P(n) &\sim n^{\alpha(q)}, \\ P(N_n) &\sim N_n^{-\alpha(q)}, \end{aligned} \quad (\text{A3})$$

with noninteger (fractional or “fractal”) values for  $\alpha(q)$ . Multifractal analysis estimates different rates by

weighting  $P_i(n)$  with a “mass” (Step 3) and examining how mass-weighted proportion grows with  $n$  (Step 4). *Step 3:* Calculate mass  $\mu_i(q, n)$ :

$$\mu_i(q, n) = \frac{[P_i(n)]^q}{\sum_{j=1}^{N_n} [P_j(n)]^q}, \quad (\text{A4})$$

where parameter  $q$  emphasizes higher or lower proportions for  $q > 1$  or  $q < 1$ , respectively.

*Step 4:* Calculate  $\alpha(q)$  as

$$\begin{aligned} \alpha(q) &= \frac{\sum_{i=1}^{N_n} \mu_i(q) \log P_i(n)}{\log n}, \\ \alpha(q) &= - \frac{\sum_{i=1}^{N_n} \mu_i(q) \log P_i(N_n)}{\log N_n}. \end{aligned} \quad (\text{A5})$$

for linear relationships between numerator and denominator,  $r \geq .95$ . If the Shannon entropy of  $\mu_i(q, n)$  yields similarly strong linear relationship with  $\log n$ , then

$$\begin{aligned} f(q) &= \frac{\sum_{i=1}^{N_n} \mu_i(q, n) \log \mu_i(q, n)}{\log n}, \\ f(q) &= - \frac{\sum_{i=1}^{N_n} \mu_i(q, n) \log \mu_i(q, n)}{\log N_n}, \end{aligned} \quad (\text{A6})$$

and  $f(\alpha(q))$  is a single point in the multifractal spectrum (Fig. 2, bottom panel).

*Step 5:* Recalculate  $f(q)$  and  $\alpha(q)$  for many values of  $q$ , spanning a range allowing correlations between numerators and corresponding denominators in Eqs. (A5) and (A6) to drop below  $r = .95$  on either end of the spectrum. We used a range  $-20 \leq q \leq 20$ , incremented by .005.

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